Benha University Faculty of Engineering – Shoubra Elec. Engineering and Control Department Duration: 2 hours		Final Exam Course: Mathematics 4 Code: EMP 202 Date : May 12 , 2018

The exam consists of one page   No. of questions: 4   Answer **All** questions   Total Mark: 40

Question 1

Solve the following equations : (12)

(a) $y` - \frac{1}{x} y = x\, e^{2x}$ (b) $(\sin x + y)\mathrm{dx} + (\mathrm{x} + \cos y)\mathrm{dy} = 0$

(c) $y`` - 4y` + 4y = 8 + e^{x}$    (d) $y`` + 4y = x^4 + x^2$

Question 2

(a)Find the L.T of : (i) $\mathrm{f(t) = e^{2t} + t.\sinh 3t}$ (ii) $\mathrm{f(t) = t^3 + e^{3t}.\cos t}$ (3)

(b)Find the inverse L.T of : (i) $\mathrm{F(s)} = \frac{1}{S^3} + \frac{s+1}{s^2+4}$ (ii) $\mathrm{F(s)} = \frac{1}{S^2 - 4s + 4}$ (3)

(c)By L.T, solve the equation : $y`` - 4y` + 4y = e^{2t}$, y(0) = 0 , y`(0) = 1. (4)

Question 3

(a) Using the bisection method, find a root to the equation : (4)

$f(x) = x^2 + \log x - 2 = 0$ in the interval [1, 2], number of iterations is 3.

(b)Find the curves : $y = a\, x^b$ and $y = a\, b^x$ that fit the data : (4)

(2, 3), (3, 5), (4, 4), (5, 10), (6, 20)

Question 4

(a)Find the integrals : (i) $\int_0^2 \frac{1}{\sqrt{y+\sqrt{y}}} dy$ (ii) $\int_2^\infty \frac{x}{1+x^5} dx$ (4)

(b)Find $f`(3)$ where $f(x) = \begin{cases} x^2 + 1, & x > 3 \\ 2^x + 2, & x \le 3 \end{cases}$ and h = 0.1 (2)

(c)If x is random variable and $f(x) = \begin{cases} k(x^2 - 2x), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$ (4)

Find the value of k such that f(x) is p.d.f and find the probability function F(x).

Good Luck   *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a) It is linear. Then $\rho = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

Then the solution is : $y.\frac{1}{x} = \int \frac{1}{x}.x\, e^{2x}dx = \int e^{2x}\, dx = \frac{1}{2}e^{2x} + c$

(b)It is exact because $p_y = 1 = q_x$.

Then $\int (y + \sin x)dx = xy - \cos x$ and $\int (x + \cos y)dy = xy + \sin y$

Then the solution is : $xy - \cos x + \sin y = c$

(c)The A.E is $m^2 - 4m + 4 = 0$. Then $m = 2, 2$

Then $y_{c.f} = A\, e^{2x} + B\, x\, e^{2x}$

Also, $y_{P.I} = \frac{1}{D^2-4D+4}(8 + e^x) = \frac{8}{0-0+4} + \frac{1}{1-4+4}e^x = 2 + e^x$

The solution is : $y = y_{c.f} + y_{P.I}$

(d) The A.E is $m^2 + 4 = 0$. Then $m = 2i,\ m = -2i$

Then $y_{c.f} = A\cos 2x + B\sin 2x$

Also, $y_{P.I} = \frac{1}{D^2+4}(x^4 + x^2) = \frac{1}{4}\left(1 + \frac{1}{4}D^2\right)^{-1}(x^4 + x^2)$

$$= \frac{1}{4}\left(1 - \frac{1}{4}D^2 + \frac{1}{16}D^4 + \cdots\right)(x^4 + x^2)$$

$$= \frac{1}{4}\left((x^4 + x^2) - \frac{1}{4}(12x^2 + 2) + \frac{24}{16}\right)$$

The solution is : $y = y_{c.f} + y_{P.I}$

--*12-Marks*

Answer of Question 2

(a)(i) $F(s) = \frac{1}{s-2} - \left(\frac{3}{s^2-9}\right)' = \frac{1}{s-2} + \frac{6s}{(s^2-9)^2}$

(ii) $F(s) = \frac{3!}{s^4} + \frac{s-3}{(s-3)^2+1}$

--*3-Marks*

(b) (i) $F(s) = \frac{1}{s^3} + \frac{s}{S^2+4} + \frac{1}{S^2+4}$. Then $f(t) = \frac{1}{2!}t^2 + \cos 2t + \frac{1}{2}\sin 2t$

(ii) $F(s) = \frac{1}{S^2-4s+4} = \frac{1}{(s-2)^2}$. Then $f(t) = t\,e^{2t}$

---*3-Marks*

(c)Since $L\{y`` - 4y` + 4y\} = L\{e^{2t}\}$, $y(0) = 0$ , $y`(0) = 1$.

Then $(s^2Y - sy(0) - y`(0)) - 4(sY - y(0)) + 4Y = \frac{1}{s-2}$

From the given condition : $s^2Y - 0 - 1 - 4Y - 0 + 4Y = \frac{1}{s-2}$

Then $(s^2 - 4s + 4)Y = \frac{1}{s-2} + 1 = \frac{s-1}{s-2}$

Then $Y = \frac{s-1}{(s-2)(s^2-4s+4)} = \frac{s-1}{(s-2)^3} = \frac{1}{(s-2)^2} + \frac{1}{(s-2)^3}$

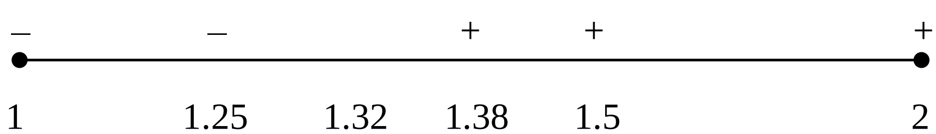
Then $y(t) = t.e^{2t} - \frac{1}{2}t^2.e^{2t}$

---*4-Marks*

<u>Answer of Question 3</u>

(a) $f(x) = x^2 + \log x - 2 = 0$

sign



Iteration	[a, b]	x_n	$f(x_n)$	sign
0	[1, 2]	1.5	0.4	+
1	[1, 1.5]	1.25	– 0.3	–
2	[1.25, 1.5]	1.38	0.04	+
3	[1.25, 1.38]	1.32		

Then $x^* = 1.32$

---*4-Marks*

(b) The curve : $y = a\,x^b = 0.85\,x^{1.55}$ and The curve : $y = a\,b^x = 1.09\,(1.57)^x$

---*4-Marks*

Answer of Question 4

(a)(i) $\int_0^2 \frac{1}{\sqrt{y+\sqrt{y}}} dy = \int_{0.1}^2 \frac{1}{\sqrt{y+\sqrt{y}}} dy = 1.46$

(ii) $\int_2^\infty \frac{x}{1+x^5} dx = \int_0^{0.5} \frac{y^2}{1+y^5} dy = 0.04$

--*4-Marks*

(b)From $f(x) = \begin{cases} x^2 + 1, & x > 3 \\ 2^x + 2, & x \le 3 \end{cases}$ and h = 0.1

Then $f`(3) = \frac{f(x+h)-f(x-h)}{2h} = \frac{f(3.1)-f(2.9)}{2(0.1)} = \frac{[(3.1)^2+1]-[2^{2.9}+2]}{2(0.1)} = 5.73$

--*2-Marks*

(c)From $\int_{-\infty}^{\infty} f(x)\, dx = \int_0^2 k(x^2 - 2x)\, dx = -\frac{4}{3}k = 1$. Then $k = -\frac{3}{4}$.

Then $f(x) = \begin{cases} -\frac{3}{4}(x^2 - 2x), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

Then $F(x) = \int_{-\infty}^{x} f(x)\, dx = \int_0^x -\frac{3}{4}(x^2 - 2x)\, dx = \begin{cases} -\frac{3}{4}\left(\frac{1}{3}x^3 - x^2\right), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

--*4-Marks*

Dr. Mohamed Eid